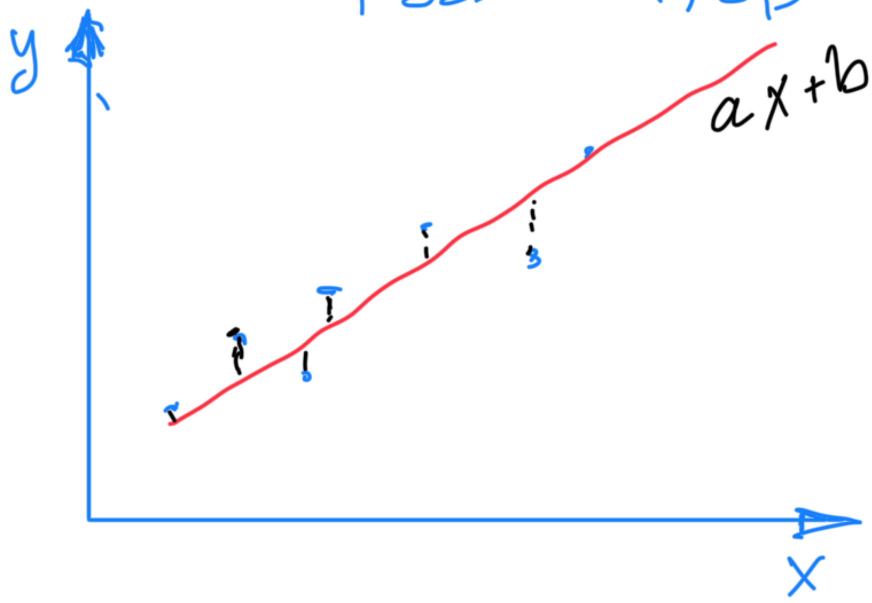
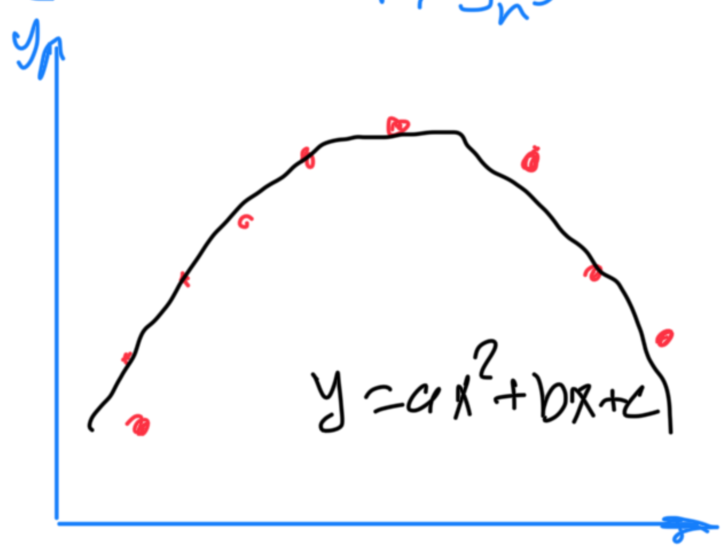
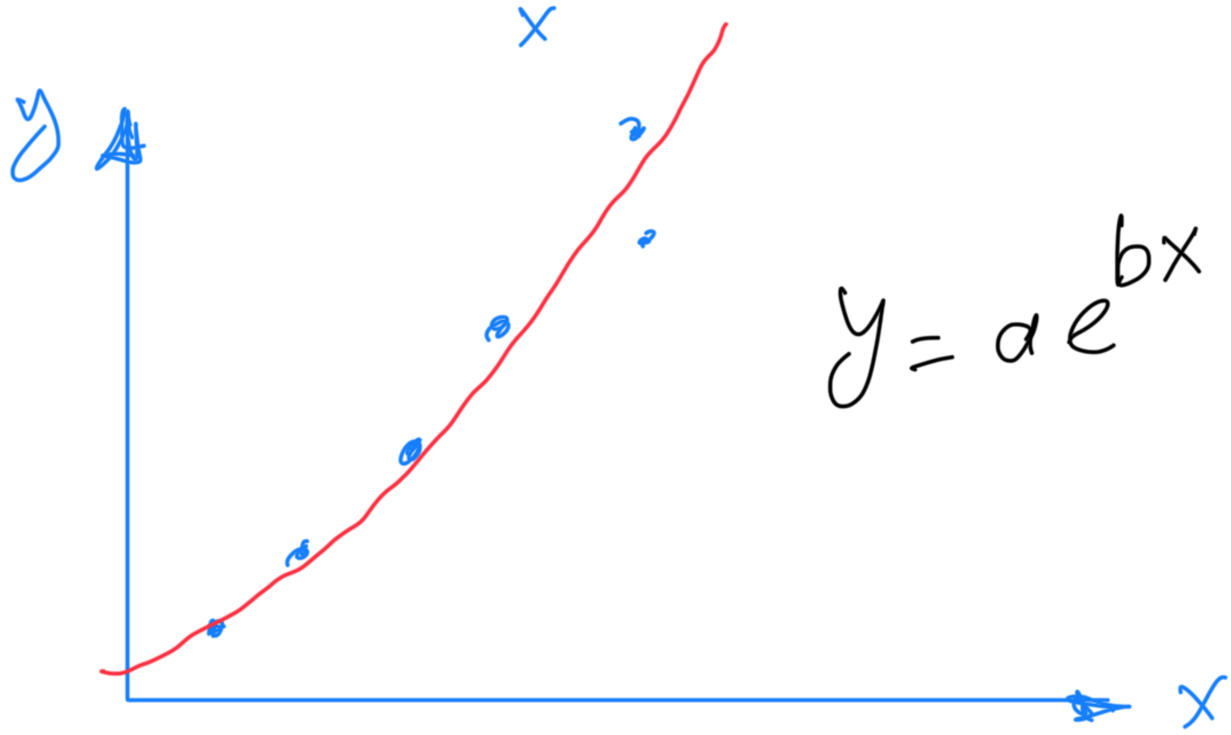
Lecture 4

Curve Fitting.

if we have The following Descrite Points $(x_0, y_0)\ (x_1, y_1) \cdots\ (x_n, y_n)$.







$$y = a e^{bx}$$

$$LSE = \sum \big(y - (ax+b)\big)^2$$

$$= \sum (y - ax - b)^2$$

to minmize This error

Diffrentiat w.r.t a & b.

$$\frac{\partial E}{\partial a} = 2\sum (y - ax - b)(-x) = 0$$

$$= \sum xy - a\sum x^2 - b\sum x \rightarrow (1)$$

$$\frac{\partial E}{\partial b} = 2\sum (y - ax - b)(-1) = 0$$

$$= \sum y - a\sum x - b\sum 1 \rightarrow (2)$$

$$a\sum x + nb = \sum y \rightarrow (3)$$

$$a\sum x^2 + b\sum x = \sum xy \rightarrow (4)$$

x	y	x^2	xy
x_0	y_0	x_0^2	$x_0 y_0$
x_1	y_1	x_1^2	$x_1 y_1$
$\vdots$	$\vdots$	$\vdots$	
x_n	y_n	x_n^2	$x_n y_n$
$\sum x$	$\sum y$	$\sum x^2$	$\sum xy$

Subbistitud in eq 3 & 4 to get The values of a & b.

Example ①

Find the least squares line for the following data

(-1, 10), (0, 9), (1, 7), (2, 5), (3, 4)

(5, 0), (6, -1)

Sol:: let the equation of the L.S.L in the form

$$y = ax + b$$

Then to find the values of a & b

$$\sum y = a \sum x + nb \rightarrow ①$$

$$\sum xy = a \sum x^2 + b \sum x \rightarrow ②$$

	x	y	x2	xy
	-1	10	1	-10
	0	9	0	0
	1	7	1	7
	2	5	4	10
	3	4	9	12
	4	3	16	12
	5	0	25	0
	6	-1	36	-6
$\sum$	20	37	92	25

Sub in eq (1) & (2) Then

$$92a + 20b = 25$$

$$20a + 8b = 37$$

by Solving these eqs we get

$$a = -1.6071$$

$$b = 8.6429$$

and The L.S.L is

$$\boxed{y = -1.6071\,x + 8.6429}$$

Example 2

Find the least squares line for

(0.5, 0.31), (1, 0.82) (1.5, 1.29)

(2, 1.85), (2.5, 2.51), (3, 3.02)

Sol:-

Let the eq of line is

$$y = ax + b$$

$$\sum y = a \sum x + nb \rightarrow (1)$$

$$\sum xy = a \sum x^2 + b \sum x \rightarrow (2)$$

Contract The table to get The required vales.

$$\sum x = 10.5$$

$$\sum y = 9.8$$

$$\sum x^2 = 22.75$$

$$\sum xy = 21.945$$

Then

$$22.75a + 10.5b = 21.945$$

$$10.5a + 6b = 9.8$$

by solving These eqs. we get

$$a = 1.096$$

$$b = -0.285$$

Then

$$\boxed{y = 1.096x - 0.285}$$

Polynomials fitting curves.

to fit the points

$(x_1, y_1), (x_2, y_2) ---- (x_n, y_n)$

in the polynomial form.

$$y = ax^2 + bx + c$$

us the following equations to find the values of a, b, & C

$$a\sum x^2 + b\sum x + nc = \sum y$$

$$a\sum x^3 + b\sum x^2 + c\sum x = \sum xy$$

$$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum x^2 y$$

Example 3

Find the least square parabola for the data.

(1, 10), (2, 20), (3, 38), (4, 60), (5, 90)

Thence find y at $x = 2.5$.

Sol let the equation of the parabola

$$y = ax^2 + bx + c$$

Then to find the values of a, b & c, use

$$a\sum x^2 + b\sum x + nc = \sum y$$

$$a\sum x^3 + b\sum x^2 + c\sum x = \sum xy$$

$$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum x_i^2 y$$

	x	y	x2	x3	x4	xy	x^2 y
	1	10	1	1	1	10	10
	2	20	4	8	16	40	80
	3	38	9	27	81	114	342
	4	60	16	64	256	240	960
	5	90	25	125	625	450	2250
$\sum$	15	218	55	225	979	854	3642

Sub in The above eqs and Solve Them to get the unknown values of

$$a = 3.1429$$
$$b = 1.1429$$
$$c = 5.6$$

Then

$$y =$$

and

$$y(2.5) = 3.1429(2.5)^2 + 1.1429(2.5) + 5.6$$

$$= \checkmark$$

Transformation for data (linearization)

We can fit other types of functions by suitable transformation to transform it to linear form.

for example;-

We can fit The data to the following

Curves.

$$y = Ce^{ax}$$

$$y = cx^{a}$$

$$y = a\ln x + b$$

and

$$y = \frac{a}{x} + b$$

Example ④

fit the curve $y = ae^{bx}$ to the following data-

x	1	2	3	4	5	6	7	8
y	15.3	20.5	27.4	36.6	49.1	65.6	87.8	117.6

Sol

$$y = ae^{bx}$$

or

$$\ln y = \ln(ae^{bx})$$

$$= \ln a + \ln e^{bx}$$

$$\ln y = \ln a + bx$$

which can be written inform

$$Y = A + bx$$

where

$$Y = \ln y$$

$$A = \ln a$$

So, we need toget the values of

$$nA + b\sum x = \sum Y$$

$$A\sum x + b\sum x^2 = \sum xY$$

x	y	$Y=\ln y$	xY	x^2
1	15.3	ln(15.3)	1. ln(15.3)	1^2
2	20.5	ln(20.5)	2. ln(20.5)	2^2
3	27.4	ln(27.4)		3^2
4	36.6			
5	49.1			
6	65.6			
7	87.8			
8	117.6	ln(117.6)	8. ln(117.6)	8^2
$\sum x$		$\sum Y$	$\sum xY$	$\sum x^2$

Then sub in the above eqs. to find the values of A & b after that

$$A = \ln a$$

Then $a = e^{A}$

and the final form of the equation can be written in the form:

$$\boxed{y = 11.4371\, e^{0.2912 x}}$$

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by the same method we can transform other curves form

for example:-

$$y = b x^{a}$$

$$\ln y = \ln b + \ln x^{a}$$

$$\ln y = \ln b + a \ln x$$

$$Y = B + a X$$

where

$$\boxed{\begin{aligned} Y &= \ln y \\ B &= \ln b \\ X &= \ln x \end{aligned}}$$

and

$$y = \frac{x}{ax+b}$$

$$\frac{1}{y} = \frac{ax+b}{x}$$

$$= a + \frac{b}{x}$$

$$Y = a + bX$$

where $Y = \frac{1}{y}$

$X = \frac{1}{x}$

Curve Fitting Sheet

1) Use least square method to fit the curve y = ax + b to the following data:

   (0, 3.2), (2, 4.1), (4, 4.2), (6, 5.4). Hence find y at x = 3.

2) Fit the curve $y = a\,x^3 + b$ to the data:

   (7.9, 0.2), (11.9, 0.4), (16.4, 0.8), (22.6, 1.6).

3) Fit the curve $y = \frac{1}{(a\,x^2 + b)}$ to the data:

   (1, 0.5), (2, 0.4), (4, 0.3), (6, 0.2), (8, 0.1). Hence find y(5.1).

4) Fit the curve $y = a\,\mathrm{Exp}(-b\,x^2)$ to the data:

   (1, 9.01), (2, 6.01), (3, 6.07), (4, 2.02), (5, 0.22), (8, 0.02).

5) Fit the curve $y = a + b\sin^2 x$ to the data:

   (0, 4.2), (π/6, 5.8), (π/4, 8.3), (π/3, 9.9), (π/2, 12.5).